ALLAN VARIANCE ESTIMATED BY PHASE NOISE MEASUREMENTS

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Abstract

For short measurement times, the Allan (or two-sample) variance can be determined from the phase noise using the mathematical conversion between time and frequency domain. This provides us with a useful tool to obtain the very short-term frequency stability (τ < 0.5 second) in the time domain, which is difficult for a time-interval counter to measure because of its resolution limitations. Before carrying out the conversion, bias from the measurement system and uncorrelated noises to the DUT (device under test), e.g. ac power noise and environmental perturbations are considered and corrected in the raw data. By doing this, the proper characteristic of the DUT seems to be revealed reasonably. If the above-mentioned bias and noises were not corrected, the generated variance would be apart from the corrected one irregularly, depending on the sampling time τ.

In this paper, both the numerical integration and the power-law model are used to practice the conversion. The numerical integration is a straightforward way to use and we can get the integral approximation easily. In addition, a common model for the phase noise is linear combinations of power law processes, which are distinguished by the integer powers (α) in their functional dependence on Fourier frequency f with the appropriate coefficients h_α. Fitting experimental data with standard regression techniques could have the values of these coefficients. Thereafter, we obtain the variance with Cutler's equation using these values. The variances from these two ways are compared and inspected. Finally, because ac power noise is always inevitably getting into the measurement system and the DUT, we also make some discussions on the role it plays in the calculations of the Allan variance.

I. INTRODUCTION

Spectral densities are measures of frequency stability in what is called the frequency domain, since they are functions of Fourier frequency. The Allan variance, on the other hand, is an example of a time domain measure. In a strict mathematical sense, the Fourier transform relation connects these two descriptions as **[1-3]**:

$$\sigma_y^{\,2}(\tau) = 2\int_0^{f_h} S_y(f)\frac{\sin^4(\pi\tau f)}{(\pi\tau f)^2}df \tag{1}$$

$S_y(f)$ is the spectral density of normalized frequency fluctuations and f_h is the high frequency cutoff of a low pass filter. In theory, if we have the information of $S_y(f)$, the Allan variance could be calculated from the above relation. Our lab has a phase noise measurement system including a FSSM100 phase noise standard (1, 5, 10, 100 MHz), a FSS1000E noise detector, a FSS1011A delay line unit, and one SRS-760

FFT (Fast Fourier Transform) spectrum analyzer. The signal reference is from a SDI LNFR-400 low-noise frequency reference with a noise level of about –173 dBc/Hz (5 MHz PM, at Fourier frequency 100 KHz). The system can measure up to –177 dBc/Hz for passive devices **[4]**. Recently, we improved the system using a cross-correlation technique, which enhances the measurement capability to measure the noise 15~20 dBc/Hz below the previous one. With $S_y(f)$ from the phase noise measurement, getting the very-short-term frequency stability ($\tau < 0.5$ sec.) in the time domain via time-frequency domain conversion is made possible. Adversely, it is not easy to measure the very-short-term frequency stability using a time-interval counter directly due to its resolution limitations. The traditional time-interval counter is applicable only when the sampling time τ is not smaller than 1 second.

As for the mathematical conversion, two methods are adopted for comparison. The numerical integration, or trapezoidal integration to be precise, could calculate the Allan variance easily after the experimental data are properly processed. Besides, the power-law model is frequently used for describing the phase noise and it assumes that the spectral density of normalized frequency fluctuations is equal to the sum of terms, each of which varies as an integer power of Fourier frequency f. Thus, there are two quantities that completely specify $S_y(f)$ for a particular power-law process: the slope on a log-log plot for a given range of f and the amplitude. The slope is denoted by α and, therefore, f^α is the straight line on a log-log plot that relates $S_y(f)$ to f. The amplitude is denoted by h_α. Therefore, $S_y(f)$ can be represented by the addition of all the power-law processes with the appropriate coefficients:

$$S_y(f) = \begin{cases} \sum_{\alpha=-2}^{+2} h_\alpha f^\alpha & \text{for } 0 < f < f_h \\ 0 & \text{for } f > f_h \end{cases} \tag{2}$$

While $2\pi f_h \tau >> 1$, Cutler derived equation (3) from equation (1) and (2):

$$\sigma_y{}^2(\tau) = h_{-2}\frac{(2\pi)^2}{6}\tau + h_{-1} 2\ln 2 + \frac{h_0}{2\tau} + h_1 \frac{1.038 + 3\ln(2\pi f_h \tau)}{(2\pi)^2 \tau^2} + h_2 \frac{3 f_h}{(2\pi)^2 \tau^2} \tag{3}$$

By properly determining the coefficients, the Allan variance with different τ could be obtained using Cutler's equation. This could be achieved by locating each particular noise process in its dominant range of f with standard regression techniques.

II. TESTS OF FIVE NOISE TYPES

Before carrying out the mathematical conversion using the experimental results, we would like to know how each noise process behaves in this conversion. According to the power-law model, there are five types of noise processes, including Random Walk FM, Flicker FM, White FM, Flicker PM and White PM, with α equal to -2, -1, 0, +1, and +2 respectively. That means we need to generate five different $S_y(f)$ for tests. For example, $S_y(f)$ of the Random Walk FM noise process could be obtained by assuming the amplitude coefficient h_{-2} equal to 1×10^{-26} with the rest equal to zero. The numerical integration and the power-law model are then used to calculate the Allan deviation (ADEV), that is, the square root of the Allan variance. With τ ranging from 1 millisecond to 10 seconds, the calculated ADEVs from both methods are shown in Figure 1 (a). It is obvious that while τ is smaller than 1 second, their ADEVs are in good agreement with each other. In this manner, $S_y(f)$ of the other noise processes can be generated by separately assuming $h_{-1} = 1\times10^{-26}$, $h_0 = 1\times10^{-26}$, $h_{+1} = 4\times10^{-28}$, and $h_{+2} = 3\times10^{-31}$ for the corresponding noise processes and their ADEVs are shown in Figure 1 (b), (c), (d), and (e).

It can be seen that for the noise processes of Flicker FM and White FM, the ADEV from both methods are in good agreement with each other while τ is smaller than 1 second; for Flicker PM, the ADEV from both methods match each other well in the whole range of τ, but for white PM, the results using the numerical integration vary up and down depending on τ, while the ones using the power-law model do not. Furthermore, only for some certain values of τ would the ADEV from both methods match each other well as to the last noise process.

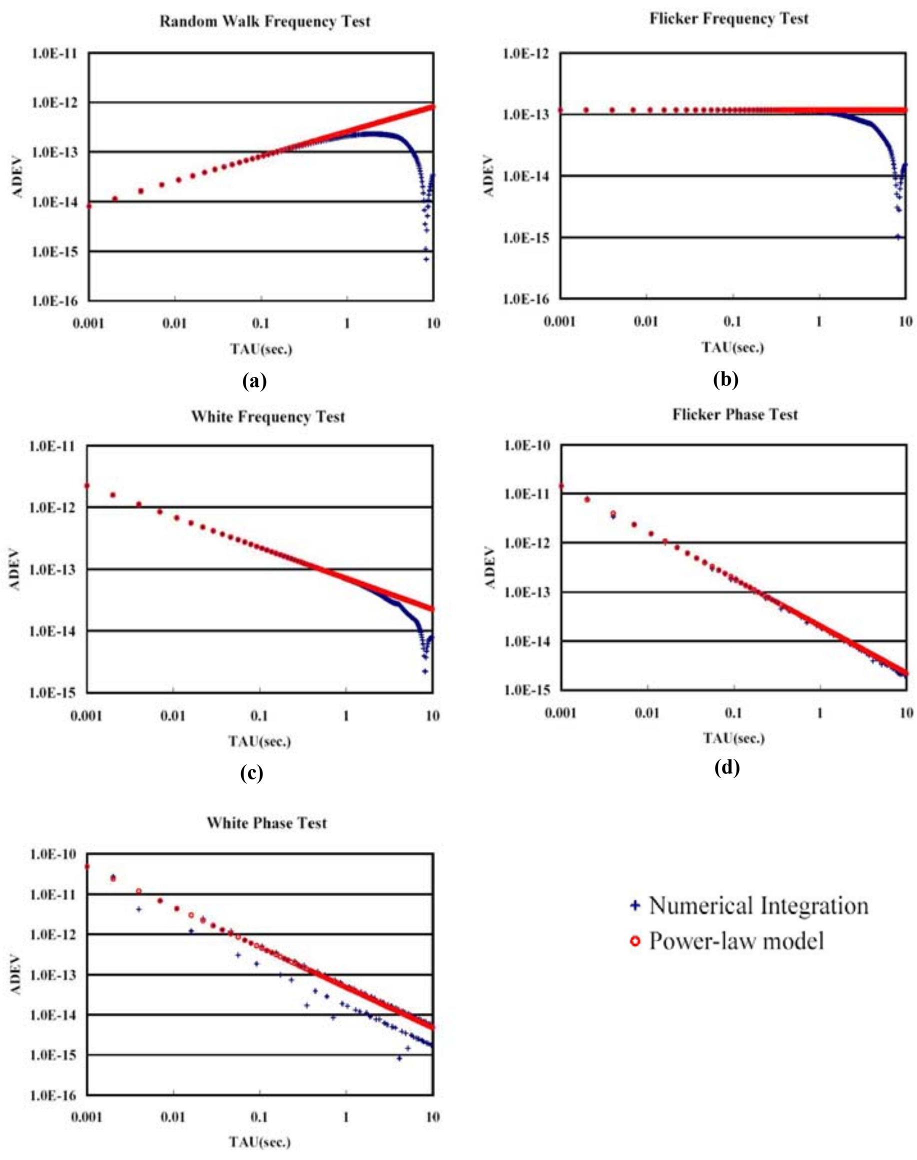

Figure 1. The ADEV of five types of noise processes using the numerical integration and the power-law model. The sampling time τ is from 1 millisecond to 10 seconds.

III. RESULTS AND CALCULATIONS

Realizing how each individual noise process behaves in the conversion using these two methods, we try to convert the real phase noise measurement data into ADEV. Here we take the noise floor of our measurement system to be an example. A 5 MHz reference signal is split with a reactive splitter to provide a pair of input signals. These signals are connected to the LO and RF ports of a double-balanced mixer. A delay line unit is required here to put the two signals into quadrature (90° out of phase) before entering the mixer. The output of the mixer is low pass filtered, amplified, and then fed into a FFT spectrum analyzer.

The spectral density of the system noise floor is shown in the upper graph of Figure 2. The Fourier frequency range is from 0.12 Hz to 99.75 kHz. There are some spur-like components appearing obviously in the graph, like noises of 60 Hz, 120 Hz, 180 Hz, etc. It is clear that most of the outliers should arise from the ac power noise, which is always inevitably getting into the measurement system or the DUT. In order to check how much these spur-like components influence the calculation of the ADEV, we use the raw data and the outliers-removed data to calculate the ADEV and then compare them. The corrected spectral density of the system noise floor is shown in the lower graph of Figure 2.

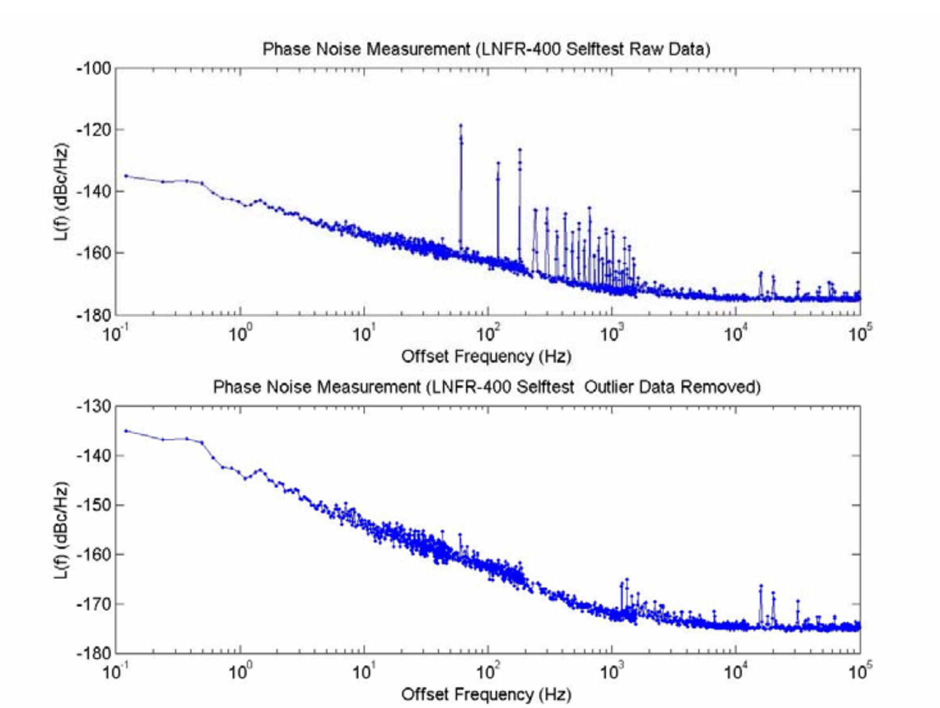

Figure 2. Spectral density of the system noise floor in Fourier frequency ranging from 0.12 Hz to 99.75 kHz. Upper graph: Spectral density consisting of spur-like periodic noises, e.g. ac power noise. Lower graph: Outliers are removed from the spectral density.

The measure L (f) on the y-axis is the prevailing expression of the phase noise among manufacturers and users of frequency standards. Its relation to $S_y(f)$ can be expressed as:

$$L(f) = \frac{1}{2} S_{\phi}(f) = \frac{1}{2}\left(\frac{v_0^{\,2}}{f^2} S_y(f)\right)$$

(4)

$S_{\varphi}(f)$ is the spectral density of phase fluctuations and v_0 is the carrier frequency. L (f) is usually reported in a dB format.

$$\frac{dBc}{Hz} = 10 \log(\mathrm{L}(f))$$

(5)

After some calculations, the results from above-mentioned data using the numerical integration are shown in Figure 3. Roughly speaking, the spur-like components make the generated ADEV bigger than the other one, but because both of them vary up and down inconsistently, depending on τ, it is hard to precisely identify how much these spur-like components contribute to the ADEV.

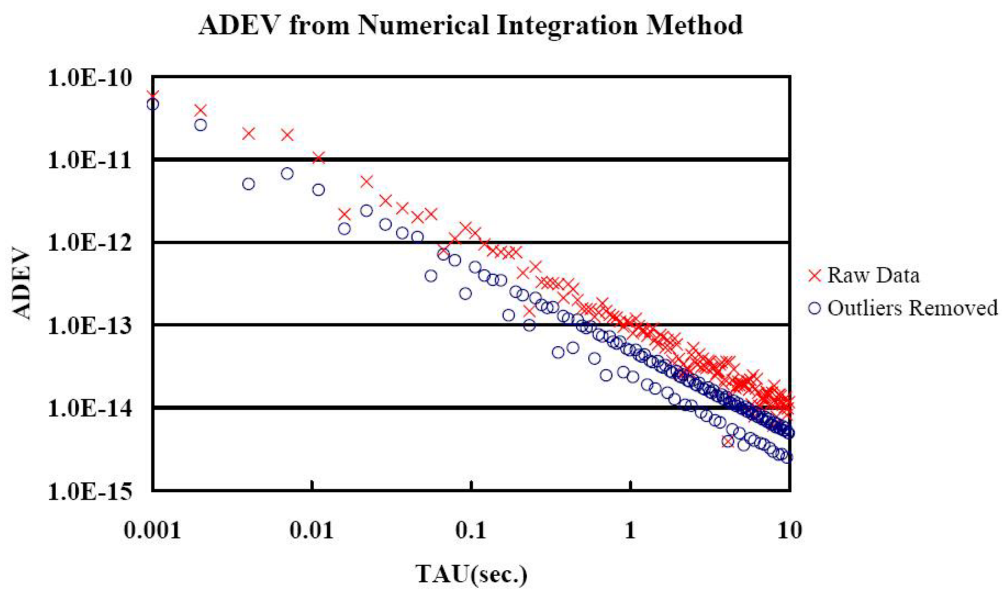

Figure 3. The ADEV calculated from the raw data with spur-like components and the outliers-removed data using the numerical integration. Both of them vary up and down inconsistently, depending on τ.

The power-law model is not applicable here because the existence of spur-like components is not in its basic assumption. It is meaningless to fit the experimental data with the model. Nevertheless, we can still make the ADEV comparison with the outliers-removed data using the numerical integration and power-law model. In the lower graph of Figure 2, we see that when f increases by one decade, L (f) also goes down by one decade for f = 0.12 ~ 1000 Hz. The dominant noise process in this range can be regarded as Flicker PM. For f = 10 ~ 99.75 kHz, L (f) is almost the same, and the dominant noise process should be White PM. We use the functions $h_{+1}f$ and $h_{+2}f^2$ to fit the data in the corresponding range and get $h_{+1} = 4.68\times10^{-28}$ and $h_{+2} = 2.61\times10^{-31}$. With the help of Cutler's equation, we can compare the generated ADEV with the one using the numerical integration, as shown in Figure 4.

We observe that the results using the numerical integration vary up and down, depending on τ, while the ones using the power-law model do not. Furthermore, only for some certain values of τ would the ADEV from both methods match each other well. Since there are only two noise processes existent in the data, it seems to be a reasonable conclusion that the noise process of White PM in the data should be responsible for this observation, as shown in the previous tests of five noise types.

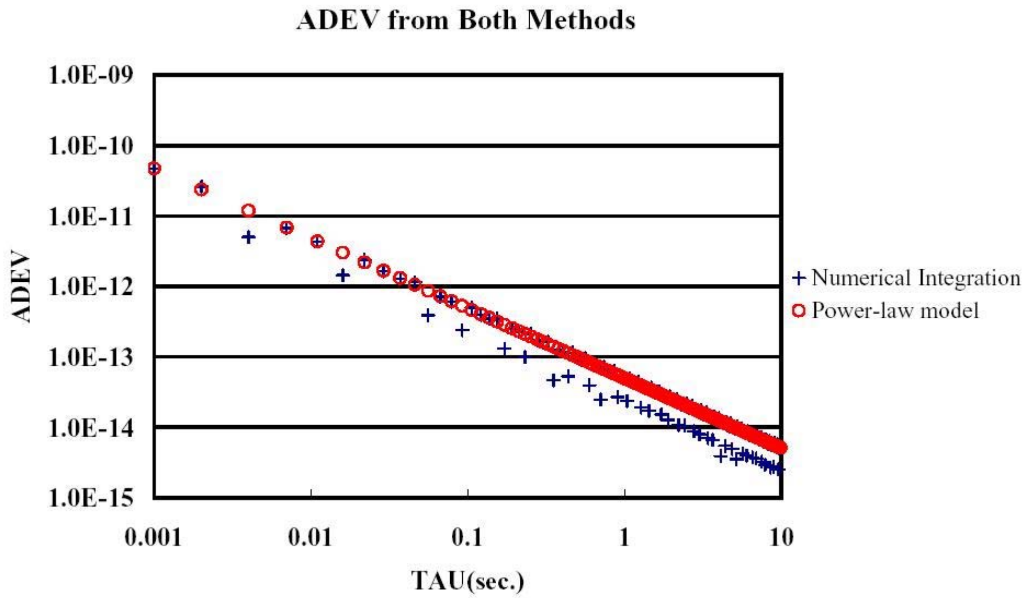

Figure 4. The ADEV calculated from the outliers-removed data using the numerical integration and the power-law model. The former varies up and down, depending on τ, while the latter does not.

IV. CONCLUSIONS

In this paper, we use the phase noise data in our lab to calculate the ADEV via the time-frequency domain conversion. The numerical integration and Cutler's equation derived from the power-law model are two methods adopted for comparison. We observe that for the noise processes of Random Walk FM, Flicker FM, White FM, and Flicker PM, the ADEV from both methods match each other well while τ is smaller than 1 second, but for White PM, the ADEV using the numerical integration varies up and down, depending on τ, while the one using the power-law model does not. This observation is shown both in the individual noise tests and the result from a real phase noise measurement. That means obtaining the very-short-term frequency stability ($\tau < 0.5$ second) in the time domain is possible if the discrepancy of the White PM calculation from the two methods can be resolved. Besides, as for the influence of ac power noise, it is hard to precisely identify how much these spur-like components contribute to the ADEV, also due to the influence of White PM using the numerical integration. In order to solve this problem, we will do more research in the near future.

REFERENCES

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[2] *"Characterization of Frequency and Phase Noise,"* Report 580 of the International Radio Consultative Committee (C.C.I.R), pp. 142-150, 1986.

[3] D. Allan, H. Hellwig, P. Kartaschoff, J. Vanier, J. Vig, G. M. R. Winkler, and N. Yannoni, 1988, *"Standard Terminology for Fundamental Frequency and Time Metrology,"* in Proceedings of the 42nd Annual Symposium on Frequency Control, 1-3 June 1988, Baltimore, Maryland, USA (IEEE 88CH2588-2), pp. 419-425.

[4] Femtosecond Systems, Inc., FSS1000E Phase Noise Detector FSS1011A Delay Line Operation Manual, Chap. 1.

QUESTIONS AND ANSWERS

SAM STEIN (Timing Solutions Corporation): I am confused about the outliers. What is the nature of the outliers?

PO-CHENG CHANG: They are the AC power influence.

STEIN: They are dispersed?

CHANG: Yes.